

Neutral bions in the \mathbb{CP}^{N-1} model for resurgence

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Abstract. Classical configurations in the \mathbb{CP}^{N-1} model on $\mathbb{R}^1 \times S^1$ is studied with twisted boundary conditions. Starting from fractional instantons with the \mathbb{Z}_N twisted boundary conditions, we review briefly the relevance of our study to resurgence phenomenon in field theory. We consider primarily configurations composed of multiple fractional instantons, called “neutral bions”, which are identified as “perturbative infrared renormalons”. We construct an explicit ansatz corresponding to topologically trivial configurations containing one fractional instanton ($\nu = 1/N$) and one fractional anti-instanton ($\nu = -1/N$), which is guaranteed to become a solution of field equations asymptotically for large separations. The attractive interactions between the instanton constituents from small to large separations are found to be consistent with the standard separated-instanton calculus. Our results suggest that the ansatz enables us to study bions and the related physics for a wide range of separations. This talk is mainly based on our recent work published in JHEP **1406**, 164 (2014) [arXiv:1404.7225 [hep-th]].

1. Introduction

Recently a progress has been brought about to understand the confinement mechanism in QCD-like theories by compactifying the theories in one spatial dimension with the period L . Because of the asymptotic freedom, the small L theory is in the weak-coupling regime ($L \ll 1/\Lambda_{\text{QCD}}$) in QCD(adj.) on $\mathbb{R}^3 \times S^1$. Instanton is a particle-like soliton of codimension D in D -dimensional spacetime. If \mathbb{Z}_N twisted boundary conditions are imposed, we can obtain fractional instantons which have fractional amount of instanton number. A molecule of fractional instantons and anti-instantons are generally called bions. For compactified space with small L , the perturbative analysis is reliable and shows that the Wilson holonomy stabilizes the \mathbb{Z}_N twisted boundary condition. In this case, one can demonstrate the condensation of classical field configurations composed of fractional instantons and anti-instantons with magnetic charge, which are called magnetic bions. Condensation of magnetic bions implies the confinement in the compactified theory [1, 2, 3, 4, 5, 6]. It is anticipated that this phenomenon continues to hold for large L , suggesting the confinement even for uncompactified theory.

It has also been pointed out that neutral bions play an important role in explaining the resurgence phenomenon [7, 8, 9, 10, 11, 12, 13, 14]. Neutral bions are composite of fractional instantons and anti-instantons with vanishing topological charge (instanton number) and zero magnetic charge. Although perturbation series in field theories are divergent, the Borel transformation is useful to obtain the re-summation of such divergent series. The Borel transform often exhibits singularities on the positive real axis producing an imaginary ambiguities in the Borel-resummed results. It has been found that these ambiguities in quantum mechanics

are cancelled by contributions from molecule of instanton and anti-instantons [15, 16, 17, 18]. However, asymptotically free field theories like QCD and the $\mathbb{C}P^{N-1}$ nonlinear sigma models exhibits similar but more serious singularities near the origin, which are called infrared renormalons [19, 20]. Recently it has been found that the nonperturbative contributions around the neutral saddle points also have imaginary ambiguities. These imaginary ambiguities of bion amplitudes are found to cancel [7, 8] precisely the imaginary ambiguities associated with the infrared renormalons, resulting in a more rigorous foundation of field theory. This phenomenon is called the resurgence. It is expected that full semi-classical expansion including perturbative and non-perturbative sectors, which is called resurgent expansion [21], leads to unambiguous and self-consistent definition of field theories in the spirit of quantum mechanical examples [15, 16, 17]. However, it is not straightforward to verify these arguments in gauge theories directly, since it is difficult to find an explicit ansatz of bion configurations.

At this stage, it is quite interesting and useful to study simpler models in lower dimensions [8, 9] instead of gauge theories in four spacetime dimensions. In particular, models in two spacetime dimensions are more tractable compared to gauge theories in four dimensions, but have many features which are common to gauge theories in four dimensions. Among them, the $\mathbb{C}P^{N-1}$ model in two spacetime dimensions has been studied as a toy model [22] of the Yang-Mills theory in four spacetime dimensions, because of their interesting features such as the dynamical mass gap, the asymptotic freedom and the existence of instantons as Bogomol'nyi-Prasad-Sommerfield (BPS) solutions [23]. The $\mathbb{C}P^{N-1}$ model on $\mathbb{R}^1 \times S^1$ with twisted boundary conditions admits BPS fractional instantons as configurations with the minimal topological charge [24, 25, 26, 27] (see also Refs. [28, 29, 30]). In Ref. [8], generic arguments on bion configurations were given in the $\mathbb{C}P^{N-1}$ model on $\mathbb{R}^1 \times S^1$ with \mathbb{Z}_N twisted boundary conditions, based on the independent instanton description taking account of interactions between far-separated fractional instantons and anti-instantons. More recently, exact non-BPS solutions were found in the $\mathbb{C}P^{N-1}$ model on $\mathbb{R}^1 \times S^1$ with the \mathbb{Z}_N twisted boundary condition [9]. The simplest non-BPS solution that they found is a four-instanton configuration composed of two fractional instantons (instanton charge $\nu = 1/N$) and two fractional anti-instantons ($\nu = -1/N$) for $N \geq 3$ placed at particular relative positions. Their study suggests that it is likely that bion configurations needed to understand the resurgence phenomenon may not be solutions of field equations. It has been known that field configurations other than the solution of the equations of motion may give significant contributions in the functional integral. Recently we constructed Ansatz of bion configurations that are guaranteed to become solutions of the field equations asymptotically as separations between constituent fractional instantons get larger, and studied interactions of constituent fractional instantons in bion configuration [31]. More recently, we have also studied bions in the Grassmann sigma models which include the $\mathbb{C}P^{N-1}$ model as a subclass [32].

2. $\mathbb{C}P^{N-1}$ model

Since bions in nonlinear sigma models in two spacetime dimensions are interesting and more tractable, let us consider the $\mathbb{C}P^{N-1}$ model. To describe the target space $\mathbb{C}P^{N-1} = \frac{SU(N)}{SU(N-1) \times U(1)}$, we denote an N -component vector of complex scalar fields as $\omega(x)$, and a normalized complex N -component vector composed from ω as $n(x)$

$$n(x) \equiv \omega(x)/|\omega(x)|, \quad |\omega| = \sqrt{\omega^\dagger \omega}. \quad (1)$$

The Lagrangian in Euclidean two dimensions for the nonlinear sigma model with $\mathbb{C}P^{N-1}$ as the target space is given by

$$\mathcal{L} = \frac{1}{2\pi g^2} (D_\mu n)^\dagger (D_\mu n), \quad (2)$$

and the topological charge representing $\pi_2(\mathbb{C}P^{N-1}) \simeq \mathbb{Z}$ is defined by

$$Q = \frac{1}{2\pi} \int d^2x \, i\epsilon_{\mu\nu} (D_\mu n)^\dagger (D_\nu n) = \frac{1}{2\pi} \int d^2x \epsilon_{\mu\nu} \partial_\mu A_\nu, \quad (3)$$

where $d^2x \equiv dx_1 dx_2$ and $\mu, \nu = 1, 2$. Our convention of the covariant derivative is $D_\mu = \partial_\mu - iA_\mu$ with a composite gauge field

$$A_\mu(x) \equiv -in^\dagger \partial_\mu n. \quad (4)$$

We consider $\mathbb{R}^1 \times S^1$ as the geometry of base manifold, and configurations on it satisfying periodicity in the x_2 direction with the period L . The Lagrangian \mathcal{L} and topological charge Q can be expressed in terms of the projection operator $\mathbf{P} \equiv nn^\dagger = \frac{\omega\omega^\dagger}{\omega^\dagger\omega}$ and using the complex coordinate $z \equiv x_1 + ix_2$,

$$\mathcal{L} = \frac{2}{g^2} \text{Tr} [\partial_z \mathbf{P} \partial_{\bar{z}} \mathbf{P}], \quad (5)$$

$$Q = 2 \int d^2x \text{Tr} [\mathbf{P} (\partial_{\bar{z}} \mathbf{P} \partial_z \mathbf{P} - \partial_z \mathbf{P} \partial_{\bar{z}} \mathbf{P})]. \quad (6)$$

We define the euclidean energy density (Lagrangian) $s(x_1)$, and the topological charge density $q(x_1)$ as functions of x_1 after the integration over x_2 :

$$s(x_1) = \frac{1}{g^2\pi} \int dx_2 \text{Tr} [\partial_z \mathbf{P} \partial_{\bar{z}} \mathbf{P}], \quad (7)$$

$$q(x_1) = \frac{1}{\pi} \int dx_2 \text{Tr} [\mathbf{P} (\partial_{\bar{z}} \mathbf{P} \partial_z \mathbf{P} - \partial_z \mathbf{P} \partial_{\bar{z}} \mathbf{P})]. \quad (8)$$

We can perform the Bogomol'nyi completion to obtain the Bogomol'nyi bound [33] for the total energy S

$$S = \frac{1}{2\pi g^2} \int d^2x \left(\frac{1}{2} |D_\mu n \mp i\epsilon_{\mu\nu} D_\nu n|^2 \pm i\epsilon_{\mu\nu} (D_\nu n)^\dagger (D_\mu n) \right) \geq \pm \frac{Q}{g^2}. \quad (9)$$

The Bogomol'nyi bound with the upper sign is saturated when the following BPS equation is satisfied

$$D_1 n - iD_2 n = \frac{1}{2} D_{\bar{z}} n = 0. \quad (10)$$

The solution of the BPS equation is precisely holomorphic $\omega(z)$. The Bogomol'nyi bound with the lower sign is saturated when the following anti-BPS equation is satisfied

$$D_1 n + iD_2 n = \frac{1}{2} D_{\bar{z}} n = 0. \quad (11)$$

The solution of the anti-BPS equation is anti-holomorphic $\omega(\bar{z})$.

In the following we omit the coupling $1/g^2$ for simplicity unless stated otherwise.

3. Borel resummation

Let us give a brief review of resurgence and the importance of bion contributions. It has been known that the number of Feynman diagrams in quantum field theory grows factorially. This factorially divergent perturbation series of quantum field theory is usually discussed by means

of the Borel transform. The Borel transform method is applicable to the following class of divergent series (called Gevrey-1)

$$P(g^2) = \sum_{q=0}^{\infty} a_q (g^2)^q, \quad |a_q| \leq C q! \left(\frac{1}{A} \right)^q, \quad (12)$$

where C, A are constants. The Borel transform $BP(t)$ is defined as

$$BP(t) = \sum_{q=0}^{\infty} \frac{a_q}{q!} t^q, \quad (13)$$

and the Borel resummation $\mathbb{B}(g^2)$ is defined as

$$\mathbb{B}(g^2) = \int_0^{\infty} \frac{dt}{g^2} e^{-t/g^2} BP(t). \quad (14)$$

One can easily see that the Borel resummation $\mathbb{B}(g^2)$ reproduces the original sum $P(g^2)$ correctly whenever one can exchange the integral and the sum. Otherwise, we need to define the sum in terms of the Borel resummation.

As a simplified toy model, let us consider a factorially divergent series of the following one with alternating signs

$$P(g^2) = C \sum_{q=0}^{\infty} q! \left(\frac{-g^2}{A} \right)^q. \quad (15)$$

Then the Borel transform becomes an analytic function without singularities on the positive real axis

$$BP(t) = C \sum_{q=0}^{\infty} \left(\frac{-t}{A} \right)^q = \frac{CA}{A+t}. \quad (16)$$

Therefore the Borel resummation is well-defined as an integral along the positive real axis

$$\mathbb{B}(g^2) = \int_0^{\infty} \frac{dt}{g^2} e^{-t/g^2} \frac{CA}{A+t}. \quad (17)$$

This alternating factorially divergent series is a typical example of Borel summable divergent series.

On the other hand, if perturbation series is not alternating, the factorially divergent series gives the Borel transform with singularities on positive real axis and the Borel resummation has imaginary ambiguities. For instance, suppose that the perturbation series $P_{\text{pert}}(g^2)$ gives non-alternating factorially divergent series like

$$P_{\text{pert}}(g^2) = C \sum_{q=0}^{\infty} q! \left(\frac{g^2}{A} \right)^q. \quad (18)$$

The Borel transform has a singularity on positive real axis

$$BP_{\text{pert}}(t) = C \sum_{q=0}^{\infty} \left(\frac{t}{A} \right)^q = \frac{CA}{A-t}, \quad (19)$$

$$\mathbb{B}_{\text{pert}}(g^2) = \int_0^{\infty} \frac{dt}{g^2} e^{-t/g^2} \frac{CA}{A-t}. \quad (20)$$

Therefore the Borel resummation has imaginary ambiguities depending on the choice of integration contours to avoid the singularity (equivalently the analytic continuation from negative real axis)

$$\mathbb{B}_{\text{pert}}(g^2 \pm \epsilon) = \text{Re}\mathbb{B}_{\text{pert}}(|g^2|) \pm i\text{Im}\mathbb{B}_{\text{pert}}(|g^2|), \quad \text{Im}\mathbb{B}_{\text{pert}}(|g^2|) \sim -\pi e^{-A/g^2}. \quad (21)$$

This ambiguous imaginary part has to be cancelled by contributions from other saddle points in the path-integral. In quantum mechanics or scalar field theories, the position of the singularity in the Borel plane has been found to correspond to molecules of instanton and anti-instanton [15, 16, 17]. However, the asymptotically free theories like QCD and the \mathbb{CP}^{N-1} model give additional singularities due to IR renormalons which are much closer to the origin and hence give much larger contributions. Recent study [7, 8] showed that the bion amplitudes give the nonperturbative contribution needed to cancel the imaginary ambiguity due to the IR renormalons.

4. Fractional instantons and neutral-bion in \mathbb{Z}_N twisted boundary conditions

When one space direction is compactified to S^1 with the circumference L , there is a possibility to impose non-periodic boundary condition. Moreover, asymptotic free theory becomes weakly coupled, and the stable configurations of Wilson holonomies of $U(1)^{N-1}$ subgroup of $SU(N)$ can be analyzed perturbatively [8, 9]. It turns out that the following \mathbb{Z}_N twisted boundary condition is most favorable under certain conditions

$$\omega(x_1, x_2 + L) = \Omega \omega(x_1, x_2), \quad \Omega = \text{diag.} \left[1, e^{2\pi i/N}, e^{4\pi i/N}, \dots, e^{2(N-1)\pi i/N} \right]. \quad (22)$$

This \mathbb{Z}_N twisted boundary condition corresponds to the vacuum with the symmetry breaking $SU(N) \rightarrow U(1)^{N-1}$. The Wilson-loop holonomy in the compactified direction is given by

$$\langle A_2 \rangle = (0, 2\pi/N, \dots, 2(N-1)\pi/N), \quad (23)$$

where the gauge fields A_2 is the composite gauge field defined in Eq. (4) of the \mathbb{CP}^{N-1} model.

The simplest solution of the BPS equation (10) satisfying the \mathbb{Z}_N twisted boundary condition in Eq. (22) is given by

$$\omega_L = \left(0, \dots, 0, 1, \lambda e^{+2\pi z/N}, 0, \dots \right)^T, \quad \omega_R = \left(0, \dots, 0, 1, \lambda e^{-2\pi z/N}, 0, \dots, 0 \right)^T. \quad (24)$$

One can easily see that these BPS solutions give the total action S and the topological charge Q as

$$S = 1/N, \quad Q = \pm 1/N. \quad (25)$$

Therefore we call this BPS solution as a fractional instanton. The fractional instanton is located at $x_1 = \frac{N}{2\pi} \log \frac{1}{\lambda}$.

A neutral bion is a molecule of fractional instantons and anti-instantons and is unstable under the annihilation process. Then we anticipate that generic neutral bion configuration is not a solution of field equations. Therefore we wish to construct a field configuration that reduces to a solution at least for far-separated fractional instantons and anti-instantons asymptotically.

From the BPS solution in Eq.(24) and their complex conjugates, we are naturally led to consider the following ansatz for the \mathbb{CP}^1 model satisfying the \mathbb{Z}_N twisted boundary condition in Eq. (22)

$$\omega = \left(0, \dots, 0, 1 + \lambda_2 e^{2\pi(z+\bar{z})/N}, \lambda_1 e^{2\pi z/N}, 0, \dots, 0 \right)^T. \quad (26)$$

A fractional instanton is located at $x_1 = \frac{N}{2\pi} \log \frac{1}{\lambda_1}$, and a fractional anti-instanton is at $x_1 = \frac{N}{2\pi} \log \frac{\lambda_1}{\lambda_2}$, respectively. This configuration becomes a solution of field equations asymptotically as the separation $\frac{N}{2\pi} \log \frac{\lambda_1^2}{\lambda_2}$ goes to infinity. For $\lambda_1^2 \gg \lambda_2$, this configuration corresponds to a $1/N$ instanton ($S = 1/N$, $Q = 1/N$) and a $1/N$ anti-instanton ($S = 1/N$, $Q = -1/N$) at large separations. The total action and the net topological charge in the large-separation limit are given by

$$S = 2/N, \quad Q = 0. \quad (27)$$

The topological charge is unchanged as the separation between the fractional instanton and the anti-instanton changes. However, the total action monotonically decreases as the instantons get closer, which shows an attractive force.

5. Interactions between fractional instanton and anti-instanton

We have evaluated analytically the total action for various values of separation between the fractional instanton and anti-instanton.

In Fig. 1, we show the total action S as a function of the separation $\tau = (1/\pi) \log \lambda_1^2/\lambda_2$ for $N = 2$. We also show the attractive force defined by $F = -\frac{dS}{d\tau}$ as a function of the separation τ . For negative values, the moduli parameter τ loses the physical meaning as the separation between the fractional instanton and anti-instanton, and merely describes a deformation of the field configuration.

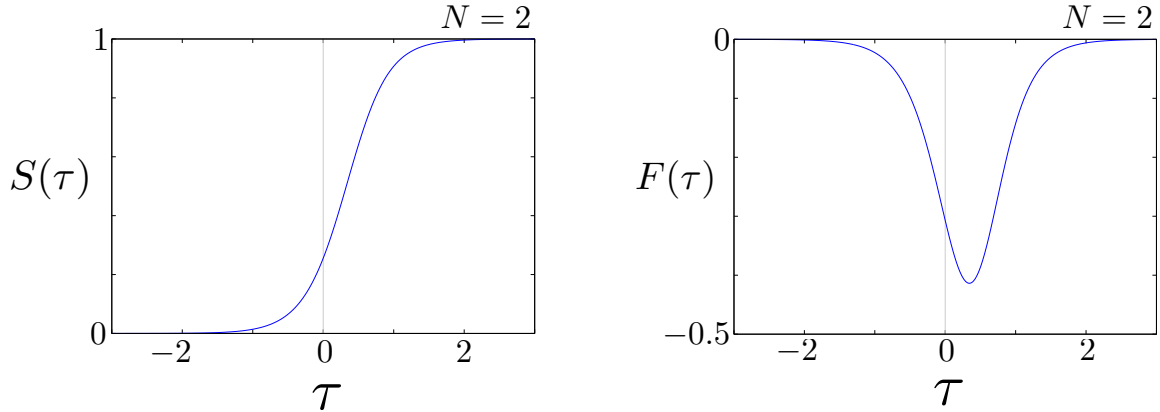


Figure 1. The $\tau = (1/\pi) \log \lambda_1^2/\lambda_2$ dependence of the total action S and the force $F = -\frac{dS}{d\tau}$ for (26) with $N = 2$. For $\tau \geq 0$, we can interpret τ as separation between the instanton constituents. The configuration is changed from $S = 1$ to $S = 0$, due to the attractive force. The configuration for $\tau \geq 1$ corresponds to neutral bions.

To compare our concrete ansatz (26) to the far-separated instanton argument in Ref. [8], we analyze the interaction part of the action for our configuration. The interaction part of the action density is written as the difference of the action density $s(x_1)$ compared to the sum of the one fractional-instanton density and one fractional-anti-instanton density $s_{\nu=1/N}(x_1) + s_{\nu=-1/N}(x_1)$,

$$s_{\text{int}}(x_1) = s(x_1) - (s_{\nu=1/N}(x_1) + s_{\nu=-1/N}(x_1)). \quad (28)$$

The total interaction action is given by integrating over the space

$$S_{\text{int}}(N, \tau) = \frac{1}{\pi} \int dx s_{\text{int}}(x_1). \quad (29)$$

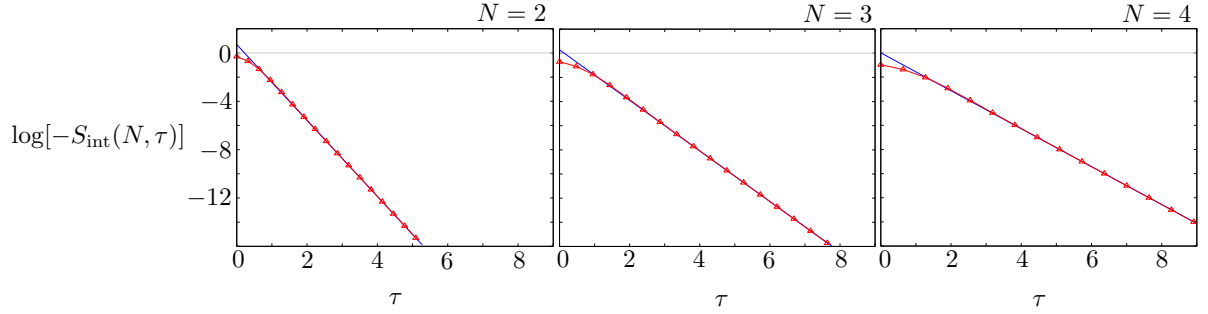


Figure 2. Plot of $\log(-S_{\text{int}}(N, \tau))$ as a function of τ for $N = 2$ (left), $N = 3$ (center) and $N = 4$ (right) for (26) (red curves with triangle points). For $\tau > 1$, the curve is almost equivalent to $-(2\pi/N)\tau + C(N)$ (blue curves).

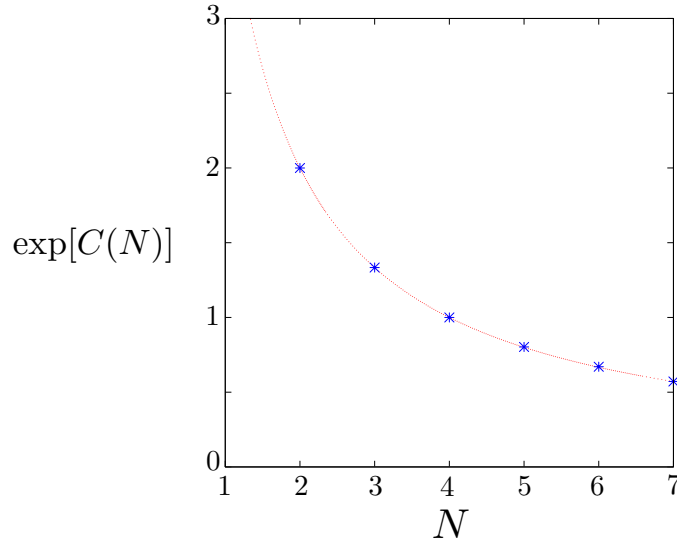


Figure 3. The coefficient of the interaction action $\exp[C(N)]$ in Eq.(32) as a function of N for $N = 2, 3, 4, 5, 6, 7$ for the Ansatz (26) (blue points). The coefficient can be approximated by $4/N$ (a red curve).

In Fig. 2, we plot the logarithm of the total interaction action $S_{\text{int}}(N, \tau)$ as a function of τ for $N = 2, 3, 4$. In the $\tau \geq 1$ region, $\log(-S_{\text{int}}(N, \tau))$ can be well approximated by analytic lines,

$$\log[-S_{\text{int}}(N, \tau)] \sim -\xi(N)\tau + C(N), \quad (\tau \geq 1), \quad (30)$$

where $\xi(N)$ is a slope and $C(N)$ is a y -intercept. In Fig. 2 we simultaneously depict these analytic lines for the three cases. The slopes ξ of the approximate lines read $\xi \sim \pi$ for $N = 2$, $\xi \sim 2\pi/3$ for $N = 3$ and $\xi \sim \pi/2$ for $N = 4$, which indicates that the slope ξ can be generally expressed as

$$\xi(N) \sim \frac{2\pi}{N}. \quad (31)$$

Therefore we observe that the interaction action can be written as the following form for $\tau \geq 1$ region,

$$S_{\text{int}}(N, \tau) \sim -e^C e^{-\xi\tau}, \quad \xi = \frac{2\pi}{N}, \quad (\tau \geq 1). \quad (32)$$

This ξ is equivalent to the (dimensionless) lowest Kaluza-Klein spectrum Lm_{LKK} , which is given as $Lm_{LKK} = |q_i - q_j| = 2\pi/N$, where q_i and q_j are two nonzero components of Wilson-loop holonomies in (23). Restoring the coupling constant g^2 , we finally obtain that the fractional instanton and anti-instanton exert attractive interaction from large to small separations as

$$S_{\text{int}}(\tau) = -\frac{8\xi}{g^2} e^{-\xi\tau}, \quad \xi \equiv \frac{2\pi}{N}. \quad (33)$$

This is precisely in accord with the approximate result for far-separated case [8].

6. Bion amplitudes

Now we are in a position to sketch briefly how our results fit into the resurgence phenomenon.

According to the study, the imaginary ambiguity arising in non-Borel-summable perturbative series is compensated by the contributions of neutral bions. This phenomenon, which is called “resurgence”, works as follows [8]: The effective interaction energy by bosonic exchange between one fractional instanton and one fractional anti-instanton is given in Eq. (33). The total bion amplitude including the fermion zero-mode exchange contribution is given by

$$\mathcal{B} \propto -e^{-2S_I/N} \int_0^\infty d\tau e^{-V_{\text{eff}}^{ij}(\tau)}, \quad (34)$$

with $V_{\text{eff}}(\tau) = S_{\text{int}}(\tau) + 2N_f\xi\tau$ and S_I being the instanton action. N_f stands for fermion flavors. For neutral bion, semiclassical description of independent fractional instantons breaks down since the interaction is attractive and instantons are merged in the end. Here, the BZJ-prescription, replacing $g^2 \rightarrow -g^2$, works to extract meaningful information from this amplitude. The prescription turns the interaction (spuriously) into a repulsive one and the amplitude becomes well-defined as

$$\mathcal{B}(g^2, N_f) \rightarrow \tilde{\mathcal{B}}(-g^2, N_f) \propto (-g^2 N/8\pi)^{2N_f} \Gamma(2N_f) e^{-2S_I/N}. \quad (35)$$

By using the analytic continuation in the g^2 complex plane, we can continue back to the original g^2 . For $N_f = 0$ case, we then encounter the following imaginary ambiguity in the amplitude as

$$\tilde{\mathcal{B}}(g^2, 0) \propto (\log(g^2 N/8\pi) - \gamma \pm i\pi) e^{-2S_I/N}. \quad (36)$$

We can rephrase this situation as follows: unstable negative modes of bions give rise to imaginary ambiguities of the amplitude. The imaginary ambiguity has the same magnitude with an opposite sign as the leading-order ambiguity ($\sim \mp i\pi e^{-2S_I/N}$) arising from the non-Borel-summable series expanded around the perturbative vacuum. The ambiguities at higher orders ($\mp i\pi e^{-4S_I/N}$, $\mp i\pi e^{-6S_I/N}$, ...) are cancelled by amplitudes of bion molecules (2-bion, 3-bion, ...), and the full trans-series expansion around the perturbative and non-perturbative vacua results in unambiguous definition of field theories.

Finally we would like to mention the results of our more recent study of bions in the Grassmann sigma model [32]. The Grassmann sigma model with the Grassmann manifold Gr_{N_F, N_C} as a target space include the $\mathbb{C}P^{N_F-1}$ model as a subclass with $N_C = 1$ (and $N_F - N_C = 1$). We have found that charged bions do not exist in the $\mathbb{C}P^{N_F-1}$ model, whereas the genuine Grassmann sigma model with $N_C \geq 2, N_F - N_C \geq 2$ admits charged

bions. The Grassmann sigma model admits BPS fractional instantons [25, 26, 27] with instanton number greater than unity (of order N_C), which cannot be reduced to composite of instantons and fractional instantons. Interaction between fractional instanton and anti-instanton in the Grassmann model is obtained to form neutral bions. We have also obtained exact non-BPS solutions of field equations for charged bions. To obtain these results, D-brane configurations are found to be a valuable tool to analyze fractional instantons and bions.

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References

- [1] M. Unsal, “Abelian duality, confinement, and chiral symmetry breaking in QCD(adj),” *Phys. Rev. Lett.* **100** (2008) 032005 [arXiv:0708.1772 [hep-th]].
- [2] M. Unsal, “Magnetic bion condensation: A New mechanism of confinement and mass gap in four dimensions,” *Phys. Rev. D* **80** (2009) 065001 [arXiv:0709.3269 [hep-th]].
- [3] M. Shifman and M. Unsal, “QCD-like Theories on $R(3) \times S(1)$: A Smooth Journey from Small to Large $r(S(1))$ with Double-Trace Deformations,” *Phys. Rev. D* **78** (2008) 065004 [arXiv:0802.1232 [hep-th]].
- [4] E. Poppitz and M. Unsal, “Conformality or confinement: (IR)relevance of topological excitations,” *JHEP* **0909** (2009) 050 [arXiv:0906.5156 [hep-th]].
- [5] M. M. Anber and E. Poppitz, “Microscopic Structure of Magnetic Bions,” *JHEP* **1106**, 136 (2011) [arXiv:1105.0940 [hep-th]].
“Seiberg-Witten and ‘Polyakov-like’ magnetic bion confinements are continuously connected,”
- [6] E. Poppitz, T. Schaefer and M. Unsal, “Continuity, Deconfinement, and (Super) Yang-Mills Theory,” *JHEP* **1210** (2012) 115 [arXiv:1205.0290 [hep-th]].
- [7] P. Argyres and M. Unsal, “A semiclassical realization of infrared renormalons,” *Phys. Rev. Lett.* **109** (2012) 121601 [arXiv:1204.1661 [hep-th]].
- [8] G. V. Dunne and M. Unsal, “Resurgence and Trans-series in Quantum Field Theory: The CP(N-1) Model,” *JHEP* **1211** (2012) 170 [arXiv:1210.2423 [hep-th]].
- [9] R. Dabrowski and G. V. Dunne, “fractionalized Non-Self-Dual Solutions in the CP(N-1) Model,” *Phys. Rev. D* **88** (2013) 2, 025020 [arXiv:1306.0921 [hep-th]].
- [10] G. V. Dunne and M. Unsal, “Generating Non-perturbative Physics from Perturbation Theory,” *Phys. Rev. D* **89** (2014) 041701 [arXiv:1306.4405 [hep-th]].
- [11] A. Cherman, D. Dorigoni, G. V. Dunne and M. Unsal, “Resurgence in QFT: Unitons, Fractons and Renormalons in the Principal Chiral Model,” *Phys. Rev. Lett.* **112** (2014) 021601 [arXiv:1308.0127 [hep-th]].
- [12] G. Basar, G. V. Dunne and M. Unsal, “Resurgence theory, ghost-instantons, and analytic continuation of path integrals,” *JHEP* **1310** (2013) 041 [arXiv:1308.1108 [hep-th]].
- [13] G. V. Dunne and M. Unsal, “Uniform WKB, Multi-instantons, and Resurgent Trans-Series,” *Phys. Rev. D* **89** (2014) 105009 [arXiv:1401.5202 [hep-th]].
- [14] A. Cherman, D. Dorigoni and M. Unsal, “Decoding perturbation theory using resurgence: Stokes phenomena, new saddle points and Lefschetz thimbles,” arXiv:1403.1277 [hep-th].
- [15] E. B. Bogomolny, “Calculation Of Instanton - Anti-instanton Contributions In Quantum Mechanics,” *Phys. Lett. B* **91**, 431 (1980).
- [16] J. Zinn-Justin, “Multi - Instanton Contributions in Quantum Mechanics,” *Nucl. Phys. B* **192**, 125 (1981).
- [17] J. Zinn-Justin and U. D. Jentschura, “Multi-instantons and exact results I: Conjectures, WKB expansions, and instanton interactions,” *Annals Phys.* **313**, 197 (2004) [quant-ph/0501136].

- [18] J. Zinn-Justin, “Perturbation Series at Large Orders in Quantum Mechanics and Field Theories: Application to the Problem of Resummation,” *Phys. Rept.* **70**, 109 (1981).
- [19] G. ’t Hooft, “Can We Make Sense Out of Quantum Chromodynamics?,” *Subnucl. Ser.* **15**, 943 (1979).
- [20] M. Beneke, “Renormalons,” *Phys. Rept.* **317**, 1 (1999) [hep-ph/9807443].
- [21] J. Ecalle, “Les Fonctions Resurgentes”, Vol. I - III (Publ. Math. Orsay, 1981).
- [22] A. M. Polyakov, “Gauge Fields and Strings,” (Contemporary Concepts in Physics, 1989) Harwood Academic Publishers, Chur and London.
- [23] A. M. Polyakov and A. A. Belavin, “Metastable States of Two-Dimensional Isotropic Ferromagnets,” *JETP Lett.* **22**, 245 (1975) [*Pisma Zh. Eksp. Teor. Fiz.* **22**, 503 (1975)].
- [24] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Instantons in the Higgs phase,” *Phys. Rev. D* **72**, 025011 (2005) [hep-th/0412048].
- [25] M. Eto, T. Fujimori, Y. Isozumi, M. Nitta, K. Ohashi, K. Ohta and N. Sakai, “Non-Abelian vortices on cylinder: Duality between vortices and walls,” *Phys. Rev. D* **73**, 085008 (2006) [hep-th/0601181].
- [26] M. Eto, T. Fujimori, M. Nitta, K. Ohashi, K. Ohta and N. Sakai, “Statistical mechanics of vortices from D-branes and T-duality,” *Nucl. Phys. B* **788**, 120 (2008) [hep-th/0703197].
- [27] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Solitons in the Higgs phase: The Moduli matrix approach,” *J. Phys. A* **39**, R315 (2006) [hep-th/0602170].
- [28] F. Bruckmann, “Instanton constituents in the $O(3)$ model at finite temperature,” *Phys. Rev. Lett.* **100**, 051602 (2008) [arXiv:0707.0775 [hep-th]].
- [29] W. Brendel, F. Bruckmann, L. Janssen, A. Wipf and C. Wozar, “Instanton constituents and fermionic zero modes in twisted CP^n models,” *Phys. Lett. B* **676**, 116 (2009) [arXiv:0902.2328 [hep-th]].
- [30] D. Harland, “Kinks, chains, and loop groups in the CP^n sigma models,” *J. Math. Phys.* **50**, 122902 (2009) [arXiv:0902.2303 [hep-th]].
- [31] T. Misumi, M. Nitta and N. Sakai, “Neutral bions in the CP^{N-1} model,” *JHEP* **1406**, 164 (2014) [arXiv:1404.7225 [hep-th]].
- [32] T. Misumi, M. Nitta and N. Sakai, “Classifying bions in Grassmann sigma models and non-Abelian gauge theories by D-branes,” arXiv:1409.3444 [hep-th].
- [33] E. B. Bogomolny, “Stability of Classical Solutions,” *Sov. J. Nucl. Phys.* **24**, 449 (1976) [*Yad. Fiz.* **24**, 861 (1976)].